

NO.6 RATIONAL EXPRESSIONS
ASSIGNMENT

Ques 1. Given - $f(x) = \frac{1}{4x^2 - 1}$

vertical
Asymptote

$$4x^2 - 1 = 0$$

$$4x^2 = 1$$

$$x^2 = \frac{1}{4}$$

$$x = \pm \frac{1}{2}$$

$$\underline{x = \frac{1}{2}, x = -\frac{1}{2}}$$

x-intercept

$$\frac{1}{4x^2 - 1} = 0 \quad (\text{Null}) \underline{\text{Ans.}}$$

y-intercept

$$y = \frac{1}{4x^2 - 1}, \text{ By putting } x = 0$$

$$y = \frac{1}{4(0)^2 - 1} = -1$$

Answer (0, -1)

Domain

$$4x^2 - 1 = 0$$

$$(2x - 1)(2x + 1) = 0$$

$$x = 1/2 \text{ or } x = -1/2$$

$$\therefore \text{Domain} = \mathbb{R} - \left\{ \frac{1}{2}, -\frac{1}{2} \right\}$$

Range

$f(x)$ should be less than or equal to -1
or $f(x)$ should be greater than 0 .

$$y = \frac{1}{4x^2 - 1}$$

$$x = \frac{1}{4y^2 - 1}$$

$$\frac{1}{x} = 4y^2 - 1$$

$$= 4y^2 = \frac{1}{x} + 1$$

$$y^2 = \frac{x+1}{4x}$$

$$y = \pm \frac{1}{2} \sqrt{\frac{x+1}{x}}$$

Range $\Rightarrow y = (-\infty, -1] \cup (0, \infty)$ Answer

Quest 2. $\frac{3x}{x^2-1} = \frac{x}{x+1} - 4$

$$\frac{3x}{x^2-1} = \frac{x - 4(x+1)}{x+1}$$

$$\frac{3x}{x^2-1} = \frac{x - 4x - 4}{x+1}$$

$$3x(x+1) = (x^2-1)(x-4x-4)$$

$$3x^2 + 3x = x^3 - 4x^3 - 4x^2 - x + 4x + 4$$

$$3x^2 + 3x = -3x^3 - 4x^2 + 3x + 4$$

$$0 = -3x^3 - 4x^2 - 3x^2 + 4$$

$$0 = -3x^3 - 7x^2 + 4$$

$$-3x^3 - 7x^2 + 4 = 0$$

$$3x^3 + 7x^2 - 4 = 0$$

$$\Rightarrow 3x^3 + 3x^2 + 4x^2 + 4x - 4x - 4 = 0$$

$$\Rightarrow 3x^2(x+1) + 4x(x+1) - 4(x+1) = 0$$

$$\Rightarrow (x+1)(3x^2 + 4x - 4) = 0$$

$$\Rightarrow (x+1)(3x^2 + 4x - 4) = 0$$

$$(x+1)(3x^2+4x-4) = 0$$

$$(x+1)(3x^2+6x-2x-4) = 0$$

$$(x+1)[3x(x+2)-2(x+2)] = 0$$

$$(x+1)(3x-2)(x+2) = 0$$

$$x+1 = 0$$

$$x = -1$$

$$3x-2 = 0$$

$$3x = 2$$

$$x = \frac{2}{3}$$

$$x+2 = 0$$

$$x = -2$$

Solution

$$x = -1, \frac{2}{3}, -2.$$

Ques 3

$$\frac{1}{(x^2-1)}$$

$$x = +1$$

$$\text{and, } x = -1$$

$$= f(x) \frac{1}{(x-1)(x+1)}$$

$$= \frac{1}{x^2-1}$$

Q4. $\frac{x+1}{2x^2-7x+6} - \frac{x-3}{2x^2-x-3}$

$$2x^2 - 7x + 6$$

$$2x^2 - 4x - 3x + 6$$

$$2x(x-2) - 3(x-2)$$

$$(2x-3)(x-2)$$

$$2x^2 - x - 3$$

$$2x^2 - 3x + 2x - 3$$

$$x(2x-3) + 1(2x-3)$$

$$(x+1)(2x-3)$$

$$\frac{x+1}{(2x-3)(x-2)} - \frac{x-3}{(x+1)(2x-3)}$$

$$\frac{(x+1)^2 - (x-3)(x-2)}{(x+1)(x-2)(2x-3)}$$

$$\frac{(x+1)^2 - (x^2 - 2x - 3x + 6)}{(x+1)(x-2)(2x-3)}$$

$$\frac{(x+1)^2 - (x^2 - 5x + 6)}{(x+1)(x-2)(2x-3)}$$

$$\frac{x^2 + 2x + 1 - x^2 + 5x - 6}{(x+1)(x-2)(2x-3)}$$

$$\frac{7x - 5}{(x + 1)(x - 2)(2x - 3)}$$

$$x + 1 = 0$$

$$x = -1$$

$$x - 2 = 0$$

$$x = 2$$

$$2x - 3 = 0$$

$$2x = 3$$

$$x = \frac{3}{2}$$

Solutions, Restrictions are

$$x \neq -1, 2, \frac{3}{2}$$
